Competition with endogenous and exogenous switching costs

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Abstract

This paper presents a general theoretical framework for dynamic competition under the presence of two types of switching costs: endogenous, which are set by providers (switching fees), and exogenous, which are specific to consumers (individual's cost of switching efforts). In a two-period game, two providers compete in prices and switching fees, and can price discriminate between old (loyal) and new (switchers) consumers.

I found there are symmetric subgame perfect equilibria in pure strategies, where the market is split equally between providers and a third of the population switch in the second period. Equilibrium prices and switching fees are not uniquely determined, but total discounted payoffs for providers and consumers are. These total payoffs are unaffected by the ability to set switching fees, but are directly affected by individual (exogenous) switching costs. Switching fees are neutral; they only intensify intertemporal price variation and, therefore, affect intertemporal payoffs by accentuating the trade-off between present and future benefits. These results explain the coexistence of competing providers that set and dismiss switching fees. They also suggest that regulatory policies should reduce individual switching costs (such as number portability, standardization or compatibility) rather than eliminate or regulate switching fees.

Keyword: Dynamic competition game, duopoly, switching costs, introductory offers, consumer heterogeneity
JEL: L11, L12, L13, L41, L42, L43

1 Introduction

Switching costs (SC) affect consumers free decision to change providers of certain product or service. These costs may be related to learning (software usage), information (medical history of patients for example), transactions (paperwork to terminate and initiate the consumption of certain service), or due to direct firms' practices to keep consumers by charging early contract termination fees (ETF) or by offering coupons and discounts to frequent consumers.

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Two types of switching costs can be distinguished: those in the form of switching fees or any other lock-in mechanism, which are set by firms; and those that are specific to individuals.[1] The latter, which is referred in the literature as exogenous SC (from the providers' viewpoint), affects directly to consumers, and providers do not set their level. These type of SC may include psychological costs of switching, learning costs, the opportunity cost of time spent on paperwork, among others, and for that reason we can think of all of these as the individual cost of switching efforts.[2]

Understanding the nature of SC and its impact on market outcomes and equilibrium conditions is important for researchers and policy makers. This given their presumed detrimental effect by increasing average prices or even changing price structures (NERA, 2003). Many have been the regulatory attempts to reduce SC; in particular, in the telecommunications industry. Thus, number portability has been widely implemented since late 90's. More recently, in Latin America, regulatory agencies targeted the sales of locked mobile handsets. Regulations also targeted switching fees; in the U.S.A. in April 2016, the FCC banned ETF in business data services; in Peru, regulations bounded ETF and restrict contracts' duration. Nonetheless, an interesting observation is the coexistence of competing providers that set ETF with those that dismiss its use.

Most of the theoretical models include only exogenous switching costs in their analysis, few as Caminal and Matutes (1990) and Shi (2013) add endogenous switching costs in the form of discounts, Chen (1997) explains poaching practices with exogenous switching costs, Fabra and Garcia (2015) and Cabral (2016) account for endogenous switching costs in their extended infinite-period models.

This paper contributes to the literature by analyzing the effect of exogenous and endogenous switching costs. It is based on Chen (1997) but differs from it by distinguishing endogenous from exogenous switching costs, adding an individual taste shock and letting the endogenous switching costs level be set in the initial period and not in the switching period (second period). It also differs from Shi (2013) by considering introductory offers and making exogenous switching cost different across consumers. My objective is to understand what determines the endogenous component (switching fee), how its strategic use affects the market outcomes and consumer welfare, why some companies would dismiss setting switching fees, and how a reduction of the exogenous component (exogenous switching costs) impacts the market. This would also allow for improved policy recommendations.

I develop a theoretical framework for a dynamic competition under the presence of switching costs, where two providers in a subscription market compete in prices and strategically use switching fees, while consumers face additionally an exogenous individual switching cost. I consider a two-period game where providers simultaneously set prices and switching fees in the first period; in the second period, providers use introductory offers since they can distinguish between old consumers and newcomers, to attract new consumers (rival's consumers).

[1] Examples of endogenous SC include the early termination fees in the telecommunications industry; the loyalty programs in the airline market; or cash rewards program in the credit card market. In this particular paper, I focus in the existence of switching fees in the form of ETF.

[2] As an example, we can think of the extra cost of getting an unlocked handset or the extra costs of unlocking a handset by a technician (locked phones are widely seen in mobile telecommunication market, and can be thought as given). Also incurred costs of security clearance paperwork relevant for the labor market in developing countries; or the time spent to get a medical examination and report to prove health condition in the insurance market.

Using a linear probability approach in a random utility setting, this model allows for heterogeneity across consumers in two dimensions, individual relative preferences and individual switching costs. Focusing on equilibria in pure strategies, I find symmetric subgame perfect equilibria, where the market is split according to consumers taste parameters in the first period, and third of the population switches in the second period. In the extended version, where providers invest on influence tastes, the market is split equally between providers. Equilibrium tuples of prices and switching fees are not uniquely determined, but providers' profits and consumers' payoffs are.

An important result is that switching fees are neutral; the presence of switching fees (endogenous to the providers) only impact intertemporal payoffs with countervailing effects, leaving multi-period payoffs unaffected. This actually would explain the observation of companies dismissing ETF, which also agrees with the finding of Cullen et al. (2016).

Another important finding is that second-period prices are increasing in individual switching costs, and loyal consumers are charged higher than newcomers (switchers). Moreover, since both, social welfare and consumer surplus are negatively affected by the exogenous individual switching cost parameter, the model suggests that lowering exogenous switching costs (by a regulatory change for example) would lead to higher consumer surplus and bigger social welfare.

This paper is organized as follows: Section 2 presents the related literature, Section 3 presents the model in detail, Section 4 presents the equilibrium analysis, and Section 6 presents the conclusions.

2 Related literature

Switching costs are usually studied by dynamic price models, and modeled by two-period models (Klemperer, 1983, 1987b,a; Farrel and Shapiro, 1988; Caminal and Matutes, 1990; Beggs and Klemperer, 1992; Padilla, 1995; To, 1996; Shaffer and Zhang, 2000), many repeated finite periods and infinite period models (Taylor, 2003; Dube et al., 2009; Pearcy, 2011; Cabral, 2012; Arie and Grieco, 2014; Fabra and Garcia, 2015; Cabral, 2016). Some explain market entry under the presence of switching costs (Klemperer, 1988; Farrel and Shapiro, 1988; Beggs and Klemperer, 1992; Wang and Wen, 1998), but most of them considerate only one type of switching costs, either exogenous or endogenous similar across consumers. Recently, Biglaiser et al. (2013, 2016) accounted for consumer heterogeneity in terms of switching costs, but they abstract from the presence of endogenous switching costs.

The effect of switching costs on competition is ambiguous. Early literature shows that switching costs increase average prices and profits, basically due to a “bargain-then-ripoff" strategy; while recent literature shows that low switching costs can be pro-competitive, they may give providers short-term incentives to lower prices and profits.

Models that include switchers and a replacement rate of established consumers (Farrel and Shapiro, 1988; Padilla, 1995; To, 1996; Cabral, 2012), in general solve for Markov perfect equilibria and get similar results. Farrel and Shapiro (1988) finds that incumbents supply only to their loyal/attached consumers and the entrants serve the newcomers. However, switching costs generate excessive entry, which creates inefficiencies in the market. In Padilla (1995), switching costs generate higher prices and profits in every period, and prices increase with firms' customer base, which also implies more difficulties in sustaining tacit collusion. [3]

Fudenberg and Tirole (2000) and Chen (1997) focuses more on the strategies used by firms to attract customers from their competitors. Toolsema (2009) adds an interesting approach by differentiating intra and interfirm switching costs, but she restricts her analysis to a static monopoly pricing structure. Shapiro (1999) deals directly with the exclusivity of services within industries with network effects.

Although switching costs are usually discussed in dynamic models, a static approach is also used (Klemperer, 1988; Shaffer and Zhang, 2000). Klemperer (1988) analyzes firms' entry decisions in markets with switching costs. According to that model, when switching costs are unavoidable, entry is found to be socially undesirable due to the welfare losses caused by the switching costs that consumers have to face and the incumbents' output that would have been efficiently provided with no entry.

Shaffer and Zhang (2000) focus their analysis to the effect of switching costs into subscription markets under symmetric and asymmetric firms; they find that when markets are symmetric it is optimal a pay-to-switch price strategy, while when markets are asymmetric, it is more profitable use a pay-to-stay strategy. Using two-period models, Beggs and Klemperer (1992) show that switching costs lead to higher equilibrium prices and higher profits, thus markets with switching costs become more attractive to the entry of new firms, and that market shares would converge to the same rate if firms exhibit similar costs. This may be the reason why switching costs reduce demand elasticity.

By modeling a two-period economy that produces a homogeneous good, Klemperer (1987b) finds that switching costs lead to increased competition in the first period to get the larger portion of the market in order to maximize second-period rents by charging high prices top loyal consumers.[4] [5] Competition intensity, however, is reduced in the following period, when also firms produce less. Thus, welfare is expected to fall due to switching costs. In a similar study, but with differentiated goods, Klemperer (1987a) finds that the effect on competition is ambiguous for the first period, but damaging in the second period due to the firms' incentive to take advantage of their loyal established consumers.

More recently, using an infinite horizon approach, Dube et al. (2009) show a negative relationship between switching costs and prices in markets with differentiated products; their result suggest consumers becomes more valuable with switching costs, so firms would compete more aggresively to attract them. This results are shared with Fabra and Garcia (2015), who find switching costs becomes pro-competitive in the long-run when market shares tend to be symmetric, when market structure is asymmetric then switching costs lead to higher prices. Under the absence of price discrimination between loyal and non-loyal consumers, Arie and Grieco (2014) show switching costs have a larger compensating effect that lead firms to reduce prices, instead of increasing them, to attract switchers from the rival. A common assumption in these models is that either are consumers homogeneous or they face same switching costs. On that aspect, Biglaiser et al. (2013, 2016) explicitly add switching costs heterogeneity in

[3] Switching costs would make punishments less severe in collusive agreements.

[4] Firms fiercely compete for attracting customers in the first period, even when that means setting prices below costs. This happens because they would charge monopoly prices in the second period to their loyal consumers.

[5] Farrel (1986) shows that firms with larger market share in the first period charge higher prices in the second period, up to the level that the firm still gets the larger market share in the second period.

their analysis under the context of market entry. They distinguish a low and a high switching cost type of consumers, and find that a large presence of consumers with low switching costs favors the incumbent by making more difficult to the entrant to attract the valuable high switching cost type consumers.

One of the earliest references on endogenous switching costs is given by Aghion and Bolton (1987), who found that locked-in or esclusive contracts are used to restrict market entry; moreover, buyers would only engaged in transactions with the entrant if they get compensated by lower prices and they covered their switching fees. Thus, these contracts serve to extract the surplus the entrant could generate if entry would have happen. Caminal and Matutes (1990) present a duopoly model with endogenous switching costs and differentiated product. They consider pricing practices to retain customers as well pre-commitment to prices or coupons in the initial period. They find that price commitment enhances competition, while coupons shrink it. Firms would prefer switching costs to be absent, but since their next period rents depend on retained consumers, they would usually use switching costs in the form of coupons or discounts. Also based on an infinite-period model, Cabral (2012) finds conditions for switching costs to affect prices in opposite directions. According to the study, switching costs in markets already competitive strengthen the competitive behavior by intensifying competition for new customers. However in markets with lower initial competition, switching costs make the market even less competitive because the switching costs' effect on reinforcing market power of larger firms dominates.

In this paper, I consider both, exogenous and endogenous switching costs in my analysis, and consider consumer heterogeneity in two dimensions: relative taste (or preferences) and individual-specific cost of switching efforts (exogenous individual switching costs). In particular, the model is based on Chen (1997) but differs from it by adding an individual taste shock and letting the endogenous switching costs level be set in the initial period and not in the switching period (second period). It also differs from Shi (2013) by considering introductory offers and making exogenous switching cost different across consumers. As in Biglaiser et al. (2016), I deal with heterogeneous switching costs, but consider in addition the coexistence of exogenous and endogenous switching costs, add introductory offers and, in an extended version, add marketing effort to influence consumer preferences.

3 A model on dynamic competition with switching costs

I consider a two period model, where there is a unit mass of consumers, who are heterogeneous in their preferences and idiosyncratic/individual switching cost. There are two competing providers A and B, who offer substitute services to their consumers.

For simplicity, I assume that providers' marginal cost is zero.The providers operate in two periods and have the same discount rate $\delta \in (0, 1]$. A contract with provider $i \in \{A, B\}$ in period 1 is a pair (T_i, s_i), where T_i is the price a consumer has to pay for the first period unlimited service of i, and s_i is a switching fee a consumer of i will have to pay to provider i if he switches in the second period from i to j, $j \neq i$ (an early termination fee ETF). A contract in period 2 with provider $i \in \{A, B\}$ specifies a price T_{ii} to a consumer that chose i in both periods and pays for the second period unlimited service of i; and a price T_{ji} to a consumer that switched providers from j to i and pays for the second period unlimited service of i.[6]

[6]From here on, I will refer to T_{ii} and T_{ji} as the prices for loyal consumers of provider i and prices for switchers to provider i.

Finally, there are no price commitments between periods.

From the demand side, and following a discrete choice approach, consumers have per period linear indirect utility (payoff) functions, have rational expectations (they are forward-looking) and have the same discount rate $\delta \in (0, 1]$.[7] Every consumer has a per period valuation v for the service, which is assumed to be big enough so the market is covered. Also every consumer k receives a taste shock σ_k –an idiosyncratic relative preference for provider A respect to B – measured in monetary terms and revealed in the first period that last only that period; in this regard, I follow the standard literature and assume non-persistent consumers' preferences. σ_k is uniformly distributed on the interval $[-\theta_1, \theta_2]$, for $\theta_1 > 0$ and $\theta_2 > 0$, and has density function $h(\sigma_k)$. Likewise, the random variables σ_k are mutually independent. [8] Providers only know the distribution of idiosyncratic variables. Later in this paper, θ_1 and θ_2 will become decision variables of the providers.

In the second period, the source of heterogeneity comes from the presence of an individual specific exogenous switching cost or inidvidual cost of switching efforts, x_k, which is uniformly distributed on the interval $[0, \omega]$ with density function $f(x)$. Also, random variables $(x_k)_{k\in[o,\omega]}$ are mutually independent. This individual switching cost x_k is learned by the consumers at the beginning of the second period and occurs independently of the first period shock. This cost of switching efforts refers to learning costs or costs (time, money, etc.) incurred by, for instance, canceling and account or unlocking a handset in the mobile telecommunications industry; cost that are unknown in the first period. This cost shock captures the uncertainty consumers face about the future, but are considered within their expected payoff maximization.[9]

Consumers then face two types of switching costs, the endogenous one that is set by the firm and appears as a fee and it basically refers to transfers from consumers to providers; and the exogenous one that are individual specific to consumers and it refers basically to pure deadweigh losses.[10]

In the first period, after observing $((s_A, T_A), (s_B, T_B))$, every consumer chooses a provider from $\{A, B\}$. Then, given their chosen provider in the first period and the new prices in the second period, consumers decide either to stay with their provider i, or to switch to the other provider and pay a switching fee s_i, $i \in \{A, B\}$ to their previous provider and incur in additional switching costs x_k.

Timeline of the game

The game timeline is described below:

[7] I took the approach reviewed in the section 2.5 of Anderson et al. (1992), also used by Cabral (2016).

[8] This relative preference is such that if $\sigma_k \geq 0$, then a consumer likes A more than B, and if $\sigma_k < 0$ B is preferred over A.

[9] For simplicity, I focus on the analysis when this cost shock is realized in the second period, rather than in the first. If consumers would know their switching cost type at the beginning of the game, consumer's decisions may become interdependent to decisions of other consumers. Under the absence of endogenous switching costs, Biglaiser et al. (2016) show that high switching cost consumers free ride and have the incentive to buy from the provider with largest share of low-SC consumers, from which they expect lower second period prices and face higher first period price.

[10] I will show that providers' profits will depend positively on the exogenous switching cost parameter, for which it is sensible to argue they will have incentives to influence on it level, but for this moment I abstract from that situation. The model could be extended by endogeneizing the exogenous switching cost parameter, but I leave that for future research.

$t=1$		$t=2$	
Stage 1	Stage 2	Stage 1	Stage 2
Providers set flat prices (T_A,T_B) and switching fees (s_A,s_B)	Consumers learn σ_{ik}, then given prices and preferences they choose their provider	Providers set new flat prices (T_{AA},T_{BA}) and (T_{BB},T_{AB})	Consumers learn x_k, then given new prices, they decide to stay or to switch

In the first stage of the first period, providers choose simultaneously (T_A, s_A) and (T_B, s_B). In the second stage, consumers observe firms' choices and simultaneously choose provider (A, B).

In the first stage of the second period, providers choose simultaneously their prices for newcomers, T_{BA} and T_{AB}, and for loyal consumers, T_{AA} and T_{BB}. Providers do not commit to keep previous period prices, they only commit in terms of switching fees. Consumers observe the new prices, and simultaneously decide whether to stay with their providers or to switch providers.

Throughout the paper I keep this limited commitment assumption. The lack of commitment is actually sensible in my model, given that I am not accounting for quality of the service in the model, and therefore utility of consumers are not fully specified. Thus, full price commitment would not be a reliable assumption if I abstract from quality valuations.

The Payoffs

Consumers

In period 1, the payoff of a consumer k of firm A is

$$R_{1Ak} = v + \sigma_k - T_A$$

and of a consumer k of firm B is

$$R_{1Bk} = v - \sigma_k - T_B$$

In period 2, the payoff of a consumer that chose firm $i \in \{A, B\}$ in period 1 is

$$R^i_{2k} = \max\{R_{ii,k}, R_{ij,k}\}$$

where,

$$\begin{aligned} R_{ii,k} &= v - T_{ii} && \text{if the consumer chose firm } i \text{ also in the first period} \\ R_{ij,k} &= v - T_{ij} - s_i - x_k && \text{if the consumer switched providers from } i \text{ to } j \end{aligned}$$

The decision variable in the first period is given by the idiosyncratic relative taste parameter, and in the second period the decision variable is the exogenous switching cost.

Thus, the multi-period net payoff of a consumer k who chooses firm i in period 1 is

$$R^i_k = R_{1ik} + \delta E_x[R^i_{2k}] \tag{1}$$

Providers

I denote α and $(1-\alpha)$ the first period endogenous market shares of providers A and B, respectively. Likewise, in the second period, n_{ii} refers the endogenous share of consumers that consume from i in both periods, and n_{ij}, to the endogenous share of consumers that switched from i to j.[11]

In the second period, providers' profits come from loyal consumers, newcomers and the switching fee collected from switchers that left. Thus, second period profits are given by

$$\pi_{2i} = n_{ii}(\cdot)T_{ii} + n_{ji}(\cdot)T_{ji} + n_{ij}(\cdot)s_i \quad \forall i, j \in \{A, B\}$$

The first period payoffs are $\pi_{1A} = \alpha T_A$ and $\pi_{1B} = (1-\alpha)T_A$, so provider i solves:

$$\max_{T_i, s_i} \pi_i = \pi_{1i} + \delta\pi_{2i} \tag{2}$$

The game is structured such that consumers are not treated as players. Consumers only make binary choices and their behavior depend on all prices and used to define providers' payoff functions. Providers are the only strategic players. Thus, I solve this game using backward induction, so I start finding the second-period equilibrium, and then continue with the first-period equilibrium in pure strategies. It is important to note that in this paper I solely focus on equilibrium solutions in pure strategies.

3.1 The Second Period Equilibrium

Given their first period choices of provider and the second period new prices, consumers make their decision to switch or stay with their current provider.

A consumer k stays with his first-period provider if and only if his net payoff with this provider is at least as high as with the other (net of switching fee and costs), that means when $R_{ii} \geq R_{ij}$ (I omit the consumer index k for simplicity). Therefore, the probability that a consumer chooses to stay with his first period provider i is:

$$\begin{aligned} Pr[stay \;\; in \;\; i|i] &= Pr(R_{ii} \geq R_{ij}) \\ &= Pr(x \geq T_{ii} - T_{ij} - s_i) \\ &= Pr(x \geq x_i) \\ Pr[switch \;\; to \;\; j|i] &= 1 - Pr[stay \;\; in \;\; i|i] \end{aligned}$$

First, for consumers that chose A as their first-period provider, we understand they revealed their relative preference for A.[12] Within this group, a consumer will be indifferent between staying in A (staying loyal) and switching to B if x is such that $R_{AA} = R_{AB}$ and $x \in [0, \omega]$, which means

$$\begin{aligned} v - T_{AA} &= v - T_{AB} - s_A - x \\ x &= (T_{AA} - T_{AB} - s_A) \\ x &= x_A \end{aligned}$$

[11] The values of n_{ii} and n_{ij} are derived from the choice probabilities of consumers choice on staying or switching, and depend on second period prices (T_{ii} and T_{ji}), switching fees s_i, and individual cost of switching efforts (exogenous switching cost) parameter ω.

[12] I assume that in the second period consumers do not have any other preference shock neither they keep the previous period's one.

From this last equation, x_A represent the savings any consumer get by switching (the benefit of switching), while x is the exogenous idiosyncratic cost of switching. Thus, x_A represents exogenous switching cost level for the A's consumer who is indifferent between staying in A or switching to B, provided that $x_A \in [0, \omega]$. If $x_A > \omega$, then the benefit of switching surpasses the maximum idiosyncratic cost, therefore every consumer of A prefers to switch to B. If $x_A < 0$, then the benefit of switching is lower than the minimum idiosyncratic cost for an A's consumer, therefore every consumer of A prefers to stay in A. If $0 \leq x_A \leq \omega$, then consumers with idiosyncratic exogenous switching cost x above x_A will stay with A and those consumers with idiosyncratic exogenous switching cost below x_A will switch to B.

Similar analysis holds for the case of consumers that chose B as their first period provider. Hence, a consumer will be indifferent between staying and switching when $R_{BB} = R_{BA}$, thus $x_B = T_{BB} - T_{BA} - s_B$ and $x = x_B$. Thus, consumers will stay or switch given that $0 \leq x_B \leq \omega$.

In general, provided that $x_i \in [0, \omega]$, the choice probabilities times the first period market share – α for A and $(1-\alpha)$ for B– are the demands of loyal consumers and switchers from each firm.

$$
\begin{aligned}
n_{AA} &= \alpha \int_{x_A}^{\omega} \frac{1}{\omega} dx = \frac{\omega - (T_{AA} - T_{AB} - s_A)}{\omega} \\
n_{AB} &= \alpha \int_{0}^{x_A} \frac{1}{\omega} dx = \frac{T_{AA} - T_{AB} - s_A}{\omega} \\
n_{BB} &= (1-\alpha) \int_{x_B}^{\omega} \frac{1}{\omega} dx = \frac{\omega - (T_{BB} - T_{BA} - s_B)}{\omega} \\
n_{BA} &= (1-\alpha) \int_{0}^{x_B} \frac{1}{\omega} dx = \frac{T_{BB} - T_{BA} - s_B}{\omega}
\end{aligned}
$$

Therefore, taking into account the values of choice probabilities, and provided that $x_i \in [0, \omega]$, second period profits are

$$
\pi_{2A} = \alpha T_{AA} - \frac{\alpha}{\omega}(T_{AA} - T_{AB} - s_A)(T_{AA} - s_A) + \frac{(1-\alpha)}{\omega} T_{BA}(T_{BB} - T_{BA} - s_B) \tag{3}
$$

$$
\pi_{2B} = (1-\alpha) T_{BB} - \frac{(1-\alpha)}{\omega}(T_{BB} - T_{BA} - s_B)(T_{BB} - s_B) + \frac{\alpha}{\omega} T_{AB}(T_{AA} - T_{AB} - s_A) \tag{4}
$$

Profit functions are quadratic and concave in their arguments (prices), so maximizing over prices (T_{ii} and T_{ji}) we expect an interior solution.[13]

Solving for the second-period equilibrium by using the first order conditions, the following is the unique solution. These results also satisfy the second order conditions:

$$
T^*_{AA} = \frac{2}{3}\omega + s_A \tag{5}
$$

[13] The second derivatives are negative: $\frac{\partial^2 \pi_{2A}}{\partial {T_{AA}}^2} = -\frac{2\alpha}{\omega} < 0$, and $\frac{\partial^2 \pi_{2A}}{\partial {T_{BA}}^2} = -\frac{2(1-\alpha)}{\omega} < 0$. Likewise, $\frac{\partial^2 \pi_{2B}}{\partial {T_{AB}}^2} = -\frac{2\alpha}{\omega} < 0$, and $\frac{\partial^2 \pi_{2B}}{\partial {T_{BB}}^2} = -\frac{2(1-\alpha)}{\omega} < 0$

$$T_{BB}^{*} = \frac{2}{3}\omega + s_B \tag{6}$$

$$T_{BA}^{*} = T_{AB}^{*} = \frac{\omega}{3} \tag{7}$$

The equilibrium outcome for second-period prices do not depend on first-period market shares due to discriminatory pricing towards loyal consumers and switchers. These prices are increasing in the exogenous switching cost ω, and the endogenous switching fee s_i only affects the prices that loyal consumers face.[14] Indeed, loyal consumers end up paying switching fees as part of their prices.

Also, given these values, second-period shares are $n_{AA} = \frac{2}{3}\alpha$, $n_{AB} = \frac{1}{3}\alpha$, similarly, $n_{BB} = \frac{2}{3}(1-\alpha)$, $n_{BA} = \frac{1}{3}(1-\alpha)$, which clearly indicates that a third of first period consumers switches in the second period. This is a similar result to Chen (1997) for the case of paying-consumers-to-switch.

Proposition 1. *Suppose $x_A, x_B \in [0,\omega]$. For every market share α, there exists a unique Nash equilibrium of the second period subgame. Second-period prices do not depend on α, the switching fee s_i only affects the prices that loyal consumers face. Also, a third of the population switch providers.*

The equilibrium in proposition 1 is a second period Nash equilibrium in the subgame and the proof is provided in the appendix. Now, using (5), (6), and (7) into (3) and (4) profits are

$$\pi_{2A}^{*} = \frac{\omega}{9}(1+3\alpha) + \alpha s_A \tag{8}$$

$$\pi_{2B}^{*} = \frac{\omega}{9}(1+3(1-\alpha)) + (1-\alpha)s_B \tag{9}$$

As expected, second period profits depend heavily in their first market share, which may imply higher incentives of providers to lock-in consumers with higher switching fees. It is easy to check that $\frac{\partial \pi_{2A}^{*}}{\partial \alpha} = \frac{\omega}{3} + s_A > 0$ if $s_A \geq -\frac{\omega}{3}$.
On the other hand, it is easy to verify that $x_i^{*} \in [0,\omega]$, and as I will show later, in equilibrium x_A and x_B lie always in the interval $[0,\omega]$.

3.2 The First Period

In the first period, consumers make a choice between providers, therefore, the payoff of a consumer k will be given by :

$$R_{1Ak} = v + \sigma_k - T_A$$
$$R_{1Bk} = v - \sigma_k - T_B$$

where σ_k is the relative preference for firm A respect to firm B , and is uniformly distributed on the interval $[-\theta_1, \theta_2]$.

In the first period, consumers take decisions based on heir multiperiod payoffs. Thus, each consumer compare R^A vs. R^B

[14] Second period prices are positively affected by exogenous switching cost: $\frac{\partial T_{ii}}{\partial \omega} = \frac{2}{3} > 0$ and $\frac{\partial T_{ij}}{\partial \omega} = \frac{1}{3} > 0$

$$R_A = R_{1A} + \delta E_x[R_{2A}]$$
$$R_B = R_{1B} + \delta E_x[R_{2B}]$$

Therefore, [15]

$$R_A = v + \sigma_A - T_A + \delta(v - \tfrac{11}{18}\omega - s_A)$$
$$R_B = v - \sigma_A - T_B + \delta(v - \tfrac{11}{18}\omega - s_B)$$

Thus,

$$Pr[choose \;\; A] = Pr[R_A \geq R_B] = Pr[\sigma \geq \hat{\sigma}]$$

A consumer is indifferent between A and B when $\hat{\sigma} = \frac{T_A - T_B + \delta(s_A - s_B)}{2}$, hence, provided that $\hat{\sigma} \in [-\theta_1, \theta_2]$, and that $\sigma \sim U[-\theta_1, \theta_2]$ with density function $h(\sigma)$, we get the choice probabilities.
Since we have a unit mass of consumers, these probabilities actually give us the first-period market shares of the providers, therefore:[16]

$$\alpha = \int_{\hat{\sigma}}^{\theta_2} h(\sigma)d\sigma = \frac{1}{2(\theta_2 + \theta_1)}(2\theta_2 - (T_A - T_B + \delta(s_A - s_B)))$$

and,

$$(1 - \alpha) = \int_{-\theta_1}^{\hat{\sigma}} h(\sigma)d\sigma = \frac{1}{2(\theta_2 + \theta_1)}(2\theta_1 + (T_A - T_B + \delta(s_A - s_B))$$

So, first-period profits for providers are $\pi_1^A = \alpha T_A$ and $\pi_1^B = (1 - \alpha)T_B$.

Providers maximize their multiperiod profits over first period prices T_i and switching fees s_i :

$$\max_{T_A, s_A} \pi_A(T_A, T_B, s_A, s_B) = \pi_{1A} + \delta\pi_{2A}^*$$
$$\max_{T_B, s_B} \pi_B(T_A, T_B, s_A, s_B) = \pi_{1B} + \delta\pi_{2B}^*$$

I omit the detailed ex-ante multiperiod profit functions, which are quadratic in their arguments (first-period prices and switching fees).[17] Solving the system of equations, we get an interior solution, subgame perfect equilibria where optimal first period prices are [18]

$$T_A^* = \frac{2}{3}(\theta_1 + 2\theta_2) - \delta(\frac{\omega}{3} + s_A) \tag{10}$$
$$T_B^* = \frac{2}{3}(2\theta_1 + \theta_2) - \delta(\frac{\omega}{3} + s_B) \tag{11}$$

There are not unique values for the switching fees, but they are bounded according to firms' and consumers' constraints.[19] [20] Since we do not impose exit barriers for firms, and to avoid

[15]We get the expected second period payoffs using the distribution of exogenous switching costs x_k. The calculation of expected second period consumer surplus is shown in the appendix.

[16]Recall that for the indifferent consumer $\sigma = \hat{\sigma}$, so we can solve for $\hat{\sigma}$.

[17]The second derivatives are negative: $\frac{\partial^2 \pi^A}{\partial T_A{}^2} = \frac{\partial^2 \pi^B}{\partial T_B{}^2} = -\frac{1}{(\theta_1 + \theta_2)} < 0$, and $\frac{\partial^2 \pi^A}{\partial s_A^2} = \frac{\partial^2 \pi^B}{\partial s_B^2} = -\frac{\delta^2}{(\theta_1 + \theta_2)} < 0$.

[18]The assumption of having consumers and providers equally patient also guarantees the Hessian matrix of the system of equations to be negative semi-definite, sufficient condition to get an interior solution.

[19]Switching fees cannot be infinite, because otherwise consumers would not buy from the provider that set them. Let's recall that switching fees are set in the first period.

[20]If consumers are characterized by a demand function, then the consumer surplus will be quadratic in price and linear in s_i, $\forall i \in \{A, B\}$. Then, probably the switching fees and optimal prices (T_i and T_{ii}, $\forall i \in \{A, B\}$) will be uniquely determined.

they leave the market in the second period, we restrict second period profits to be at least zero. Consumers, in other hand, will have at least zero expected second period payoffs. Thus $\forall i \in \{A, B\}$

$$-\frac{\omega(2\theta_1 + 3\theta_2)}{3(\theta_1 + 2\theta_2)} \leq s_A^* \qquad \text{thus}, \pi_{2A} \geq 0 \tag{12}$$

$$-\frac{\omega(3\theta_1 + 2\theta_2)}{3(2\theta_1 + \theta_2)} \leq s_B^* \qquad \text{thus}, \pi_{2B} \geq 0 \tag{13}$$

$$s_i^* \leq v - \frac{11\omega}{18} \qquad \text{thus}, E[R_{2i}] \geq 0 \tag{14}$$

Given the boundaries for switching fees, s^{min} and s^{max}, both are decreasing in the exogenous switching cost parameter ω.[21] An increase of the exogenous switching cost parameter would displace the feasible region for switching fees at a lower level, which would imply a substitudability between exogenous and endogenous switching costs: lower exogenous switching costs imply higher upper bound for switching fees. It is important to highlight that switching fees, s_A and s_B, are not necessarily equal, but they must satisfy the above conditions.

Note that negative switching fees are equilibrium outcome where providers may raise the first period prices in return for partial money-back guarantee in case consumers leave. This requires a commitment of providers to pay, in the second period, negative s_i for each consumer, in case he leaves. Even though conditions 12 to 14 guarantee non-negative expected payoffs of providers, still they may end up with negative profit and possibly not able to make the refund. Therefore, more sensible switching fees should be restricted to non-negative values.

Second period prices are given by the following:

$$T_{AA}^* = \frac{2\omega}{3} + s_A^* \tag{15}$$

$$T_{BB}^* = \frac{2\omega}{3} + s_B^* \tag{16}$$

$$T_{BA}^* = T_{AB}^* = \frac{\omega}{3} \tag{17}$$

First-period prices are decreasing in the exogenous cost parameter ($\frac{\partial T_i^*}{\partial \omega} < 0$) and in the discount factor ($\frac{\partial T_i^*}{\partial \delta} < 0$). Second-period prices are positively affected by exogenous switching cost parameter ($\frac{\partial T_{ii}^*}{\partial \omega} = \frac{2}{3}$ and $\frac{\partial T_{ij}^*}{\partial \omega} = \frac{1}{3}$) and are not affected by the discount factor ($\frac{\partial T_{ii}^*}{\partial \delta} = \frac{\partial T_{ij}^*}{\partial \delta} = 0$). So an external reduction of exogenous switching costs would reduce second period prices, for both loyal consumers and switchers; but this reduction also would increase first period prices and both boundaries of endogenous switching fees (if the change is anticipated for the providers). Likewise, given the equilibrium prices and fees, I verify that $x_A = x_B = \frac{\omega}{3}$ which lies in the interval $[0, \omega]$ and supports proposition 1.

First period market share of A, $\alpha = \frac{\theta_1 + 2\theta_2}{3(\theta_1 + \theta_2)}$, is increasing in the taste parameter that favors it θ_2, and decreasing in θ_1 (the taste parameter that favors the rival), conversely for the case of market share of provider B. [22] Using (10) to (14) into the profit functions of the providers,

[21] They have negative partial derivatives respect to ω given that $\theta_1 > 0$ and $\theta_2 > 0$.

[22] It is easy to verify that $\frac{\partial \alpha}{\partial \theta_1} = -\frac{\theta_2}{3(\theta_1 + \theta_2)^2} < 0$ and $\frac{\partial \alpha}{\partial \theta_2} = \frac{\theta_1}{3(\theta_1 + \theta_2)^2} > 0$.

second period profits are

$$\pi^*_{2A} = \frac{1}{9(\theta_1 + \theta_2)}[\omega(2\theta_1 + 3\theta_2) + 3s_A(\theta_1 + 2\theta 2)] \tag{18}$$

$$\pi^*_{2B} = \frac{1}{9(\theta_1 + \theta_2)}[\omega(3\theta_1 + 2\theta_2) + 3s_B(2\theta_1 + \theta 2)] \tag{19}$$

first period profits are

$$\pi^*_{1A} = \frac{(\theta_1 + 2\theta 2)}{9(\theta_1 + \theta_2)}[2(\theta_1 + 2\theta 2) - \delta(\omega + 3s_A)] \tag{20}$$

$$\pi^*_{1B} = \frac{(2\theta_1 + \theta 2)}{9(\theta_1 + \theta_2)}[2(2\theta_1 + \theta_2) - \delta(\omega + 3s_B)] \tag{21}$$

and multiperiod profits are

$$\pi^*_A = \frac{\delta\omega}{9} + \frac{2(\theta_1 + 2\theta_2)^2}{9(\theta_1 + \theta_2)} \tag{22}$$

$$\pi^*_B = \frac{\delta\omega}{9} + \frac{2(2\theta_1 + \theta_2)^2}{9(\theta_1 + \theta_2)} \tag{23}$$

In this interior solution, multiperiod profits are not affected by the presence and setting of switching fees, $\frac{\partial \pi^*_i}{\partial s_i} = 0 \ \forall i \in \{A, B\}$, but profits are increasing in the exogenous switching costs parameter ω and the taste parameter that favors the provider (for instance $\frac{\partial \pi^*_A}{\partial \theta_2} > 0$ and $\frac{\partial \pi^*_A}{\partial \theta_1} < 0$). Given that profits are increasing in ω, it is sensible to think providers would have the incentives to influence in its level. That would explain, why providers would stand against any policy that aims to reduce individual exogenous SC, or even invest in increasing those costs by any means.[23]

The indifferent consumer has an idiosyncratic taste level of $\hat{\sigma} = \frac{\theta_2 - \theta 1}{3}$ and gets multiperiod payoff (expected payoff) of

$$R_i = v(1 + \delta) - \frac{5\omega\delta}{18} - (\theta_1 + \theta_2) \tag{24}$$

which is non-negative whenever $v \geq \frac{5\omega}{18}\frac{\delta}{1+\delta} + \frac{\theta_1 + \theta_2}{1+\delta}$.

Given the equilibrium outcomes, we get the following total consumer surplus function for consumers of provider A and B are:

$$CS_A = \frac{\theta_1 + 2\theta_2}{54}(18(1 + \delta) - 5\delta\omega - 3(5\theta_1 + 4\theta_2))$$

$$CS_B = \frac{2\theta_1 + \theta_2}{54}(18(1 + \delta) - 5\delta\omega - 3(4\theta_1 + 5\theta_2))$$

Therefore, the total consumer surplus is given by the equation below, which clearly indicates consumer surplus decreases with the individual exogenous switching costs parameter.

$$CS = (\theta_1 + \theta_2)\left(v(1 + \delta) - \frac{5\omega\delta}{18}\right) - \frac{13(\theta_1^2 + \theta_2^2)}{18} - \frac{14\theta_1\theta_2}{9} \tag{25}$$

[23] Creating many complex consumption plans, requiring canceling accounts at certain time in certain location only, using different standards, etc.

Whenever $v \geq \frac{5\omega}{18}\frac{\delta}{1+\delta} + \frac{\theta_1+\theta_2}{1+\delta}$, general consumer surplus (CS) decreases in θ_1 and in θ_2.[24] This indicates that consumers would be worse off with larger relative taste over providers, because providers can extract higher rents from consumer by increasing first period prices.[25]

3.3 Switching fees are neutral: comparison to benchmark and asymmetric model

Given the results presented previously, it is natural to ask whether the results change if none of the providers set switching fees and or if only one provider sets it. To answer that question, I solved the game for a benchmark model (with no switching fees), and for an alternative model when only one player set switching fees.
Benchmark model: In the scenario where the providers do not set switching fees and maximize profits only over prices, providers solve the following:

$$\max_{T_A} \pi_A(T_A, T_B) = \pi_{1A} + \delta\pi^*_{2A}$$
$$\max_{T_B} \pi_B(T_A, T_B) = \pi_{1B} + \delta\pi^*_{2B}$$

As shown in Table 1, the ex-ante multiperiod payoffs (π^*_i and R^*_i) and first period market shares, are the same as if they would have set switching fees. Switching fees only intensify the inter-temporal compensation effect of prices.

Table 1: Comparison between model without and with switching fees

Results for provider A	Benchmark model (with no switching fees) $\max_{T_i} \pi_i(T_i, T_j) = \pi_{1i} + \delta\pi^*_{2i}$	Model with switching fees $\max_{T_i,si} \pi_i(T_i, s_i, T_j, s_j)$
T^*_A	$\frac{2(\theta_1+2\theta_2)}{3} - \frac{\delta\omega}{3}$	$\frac{2(\theta_1+2\theta_2)}{3} - \delta(\frac{\omega}{3} + s^*_A)$
s^*_A	-	$-\frac{\omega(2\theta_1+3\theta_2)}{3(\theta_1+2\theta_2)} \leq s^*_A \leq v - \frac{11\omega}{18}$
T^*_{AA}	$\frac{2}{3}\omega$	$\frac{2}{3}\omega + s^*_A$
T^*_{BA}	$\frac{\omega}{3}$	$\frac{\omega}{3}$
π^*_{1A}	$\frac{(\theta_1+2\theta 2)[2(\theta_1+2\theta 2)-\delta\omega]}{9(\theta_1+\theta_2)}$	$\frac{(\theta_1+2\theta 2)[2(\theta_1+2\theta 2)-\delta(\omega+3s^*_A)]}{9(\theta_1+\theta_2)}$
π^*_{2A}	$\frac{\omega(2\theta_1+3\theta_2)}{9(\theta_1+\theta_2)}$	$\frac{\omega(2\theta_1+3\theta_2)+3s^*_A(\theta_1+2\theta 2)}{9(\theta_1+\theta_2)}$
π^*_A	$\frac{\delta\omega}{9} + \frac{2(\theta_1+2\theta_2)^2}{9(\theta_1+\theta_2)}$	$\frac{\delta\omega}{9} + \frac{2(\theta_1+2\theta_2)^2}{9(\theta_1+\theta_2)}$
R^*_A	$v(1+\delta) - \frac{5\omega\delta}{18} - (\theta_1+\theta_2)$	$v(1+\delta) - \frac{5\omega\delta}{18} - (\theta_1+\theta_2)$
α	$\frac{\theta_1+2\theta_2}{3(\theta_1+\theta_2)}$	$\frac{\theta_1+2\theta_2}{3(\theta_1+\theta_2)}$

Notes: $\theta_1 > 0$, $\theta_2 > 0$, $\delta \in (0, 1]$.

[24] Partial derivatives are $\frac{\partial CS}{\partial\theta_1} = \frac{18v(1+\delta)-5\omega\delta}{18} - \frac{13\theta_1+14\theta_2}{9}$ and $\frac{\partial CS}{\partial\theta_2} = \frac{18v(1+\delta)-5\omega\delta}{18} - \frac{14\theta_1+13\theta_2}{9}$. If $v \geq \frac{5\omega}{18}\frac{\delta}{1+\delta} + \frac{\theta_1+\theta_2}{1+\delta}$, meaning that consumers obtain non-negative multiperiod payoffs, then $\frac{\partial CS}{\partial\theta_1} < 0$ and $\frac{\partial CS}{\partial\theta_2} < 0$.

[25] Thus, providers may actually have the incentives to influence such relative taste through marketing efforts, because by stretching out their relative taste parameter (influencing by a lot their preferences respect to the other provider), they can increase first period prices and multiperiod profits.

Asymmetric model: If only one provider sets a switching fee (and the other do not), providers maximize the following ex-ante multiperiod profits.

$$\max_{T_A, sA} \pi_A(T_A, s_A, T_B) = \pi_{1A} + \delta\pi^*_{2A}$$
$$\max_{T_B} \pi_B(T_A, T_B) = \pi_{1B} + \delta\pi^*_{2B}$$

Then, the optimal payoffs are the same in all scenarios. Switching fees only intensify the intertemporal compensation through prices, and do not affect multiperiod payoffs.
The results are summarized in the following proposition.

Proposition 2. *Suppose* $v \geq \frac{5\omega}{18}\frac{\delta}{1+\delta} + \frac{\theta_1+\theta_2}{1+\delta}$.

- *While there are multiple subgame perfect equilibria in pure strategies, the ex-ante multiperiod payoffs of consumers and providers are uniquely determined, and providers obtain positive multiperiod profits.*

- *Ex-ante multiperiod payoffs of consumers and providers are unaffected by the use of switching fees. Indeed, different combinations of optimal prices and switching fees (where zero switching fee is also in the equilibria) lead to same payoffs' functions.*

- *First period market share of a provider is uniquely determined by* θ_1 *and* θ_2. *For provider B, this market share increases in* θ_1, *and for provider A, it increases in* θ_2.

- *A third of the population switch in the second period.*

The equilibria described in proposition 2, is supported by prices, profits and consumer payoff given by equations (10) to (24).

Increasing the taste parameter, either θ_1 or θ_2 will determine consumers' choice of a provider. Therefore, providers would have the incentive to invest in changing the magnitude of these preferences. The following section extends the model.

3.4 Extended model: Providers invest on marketing

Given that providers increases market share and profits with relative preference parameter, we enable providers to invest on it. Thus, assuming this 'marketing' cost to be convex, and for any $\phi > 0$, providers' first period profit changes to:

$$\pi_{1A} = \alpha T_A - \phi\theta_2^2$$
$$\pi_{1B} = (1-\alpha) T_B - \phi\theta_1^2$$

In the stage zero of period one, providers decide how much to invest to increase the relative taste that favors them. Thus, the timeline of this extended model is depicted in the following diagram.

$t=1$			$t=2$	
Stage 0	Stage 1	Stage 2	Stage 1	Stage 2
Providers invest on increasing taste preferences towards them, set level (θ_1,θ_2)	Providers set flat prices (T_A,T_B) and switching fees (s_A,s_B)	Consumers learn σ_k, then given prices and preferences they choose their provider	Providers set new flat prices (T_{AA},T_{BA}), (T_{BB},T_{AB})	Consumers learn x_k, then given new prices, they decide to stay or to switch

Second-period results still apply, but first-period results differ from previous analysis due to the introduction of convex costs of advertising. Thus, solving by backward induction, providers' marketing investment lead to equal maximum relative preferences level.[26]

$$\theta_1^* = \theta_2^* = \frac{5}{12\phi} \tag{26}$$

In the equilibria, $\hat{\sigma} = 0$ and providers A and B split the market equally, $\alpha = \frac{1}{2}$.
$\forall i \in \{A, B\}$ optimal switching fees and prices satisfy (27) to (30).

$$-\frac{5\omega}{9} \leq s_i^* \qquad \text{thus, } \pi_{2i} \geq 0 \tag{27}$$

$$s_i^* \leq v - \frac{11\omega}{18} \qquad \text{thus, } E[R_{2i}] \geq 0, R_{ij} \geq 0 \tag{28}$$

equilibrium prices are

$$T_i^* = \begin{cases} \frac{5}{6\phi} - \delta(\frac{\omega}{3} + s_i^*) & \text{if } \phi \geq \frac{15}{18v(1+\delta)-5\delta\omega} \\ v(1+\delta) - \delta(\frac{11\omega}{18} + s_i^*) & \text{otherwise} \end{cases} \tag{29}$$

$$\begin{aligned} T_{ii}^* &= \frac{2}{3}\omega + s_i^* \\ T_{BA}^* = T_{AB}^* &= \frac{\omega}{3} \end{aligned} \tag{30}$$

Given the boundaries s^{min} and s^{max}, an increase of the individual cost of switching effort (exogenous switching cost) parameter would displace the feasible region for switching fees at a lower level.[27] First period prices T_i^* in (29) are such that they guarantee non-negative first-period payoffs to consumers.

Providers make ex-ante multiperiod profits [28]

$$\pi_A^* = \pi_B^* = \frac{\delta\omega}{9} + \frac{35}{144\phi} \tag{31}$$

Second and first period profits are

$$\pi_{2i}^* = \frac{5\omega}{18} + \frac{s_i}{2}$$

[26]If we allow for different ϕ's for both providers, preferences and, therefore, first period market shares would differ according to the values of ϕ_A and ϕ_B. The magnitude and direction of such changes is left for future research.

[27]They have negative partial derivatives respect to ω, $\frac{\partial s_i^{mi}}{\partial \omega} = -\frac{5}{9} < 0$, and $\frac{\partial s_i^{ma}}{\partial \omega} = -\frac{11}{18} < 0$.

[28]The threshold levels to switch in the second period are $x_A^* = x_B^* = \frac{\omega}{3}$, which are within the interval $[0, \omega]$, so the optimal solutions are indeed in the equilibria.

$$\pi_{1i}^* = \frac{35}{144\phi} - \delta(\frac{s_i}{2} + \frac{\omega}{6}) \qquad \forall i \in \{A, B\}$$

Notice that $\frac{\partial \pi_{1i}^*}{\partial s_i} < 0$, $\frac{\partial \pi_{2i}^*}{\partial s_i} > 0$, and $\frac{\partial \pi_i^*}{\partial s_i} = 0$.

For chosen switching fees that satisfies (27) and (28), an increase in them affects positively to second-period profits but negatively to first-period profits. The effects cancel out for the ex-ante multiperiod profit of providers, their ex-ante multiperiod profits are unaffected by their choice of switching fee's levels.

Multiperiod profits are increasing in the individual cost of switching effort parameter ω and in the discount factor δ. First period profits are decreasing in these individual (exogenous) switching costs and second period profits are increasing in them.

On the other hand, for $\phi \geq \frac{15}{18v(1+\delta)-5\delta\omega}$, the indifferent consumer obtains ex-ante multiperiod payoff of

$$R_i = v(1 + \delta) - \frac{5\omega\delta}{18} - \frac{5}{6\phi} \qquad \forall i \in \{A, B\}$$

Notice also that this payoff does not depend on switching fee.

Additionally, second period market shares are $n_{ii} = \frac{1}{3}$, $n_{ij} = \frac{1}{6}$. Thus the probability to stay loyal is $\frac{2}{3}$ and the probability to switch (the share of switchers) is $\frac{1}{3}$. The results of the two-period model are summarized in the following proposition.

Proposition 3. *Suppose* $v \geq \frac{5\omega}{18}\frac{\delta}{1+\delta} + \frac{1}{6\phi(1+\delta)}$, *and* θ_1 *and* θ_2 *are choice variables.*

- *In every subgame perfect equilibrium,* $\theta_1^* = \theta_2^* = \frac{5}{12\phi}$, *and each provider obtains half of the market in the first period. In the second period, a third of the population switch.*
- *Any pair* (s_A, s_B) *satisfying (27) and (28) uniquely determines an equilibrium outcome of the game, and any equilibrium switching fee* s_i^* *satisfies conditions (27) and (28). Thus, a zero switching fee belongs to an equilibrium outcome.*
- *While any switching fee satisfying such conditions is an equilibrium outcome, the period prices (*T_i^* *and* T_{ii}^**) are uniquely determined by any choice of such switching fees. Thus, different combinations of optimal prices and switching fees lead to unique ex-ante payoffs for consumers and providers.*

Negative switching fees are possible in this model but up to a limit, s_i^{mi}. This means providers care so much on the present and to extract consumer surplus as much as possible by raising first period prices and guaranteeing partial money-back (pay to consumers) if consumers decide to leave. Once again, as mentioned before, this requires a commitment of providers to pay negative s_i to consumers that leave in the second period. [29] The feasible region remain constant across different level of discount factors, and is displaced downwards with bigger exogenous switching cost parameter (ω).

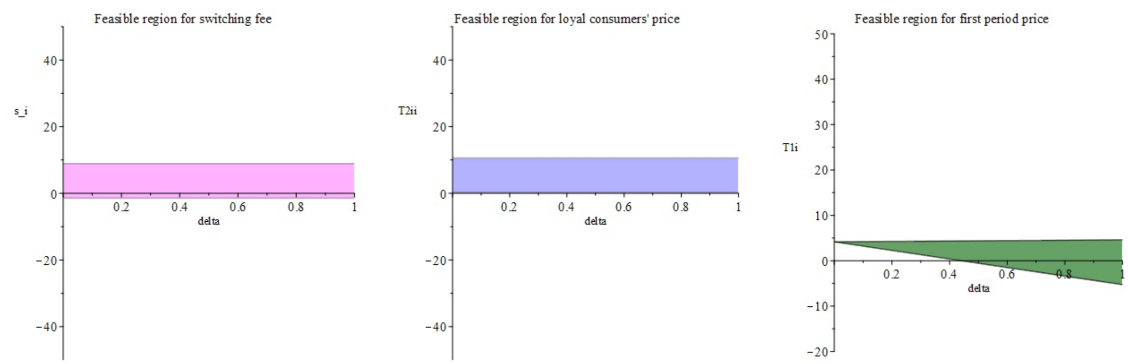
Proposition 4. *The subgame-perfect equilibrium outcome presented in proposition 3, by which a portion of consumers switch, is unique.*

[29] Despite conditions 27 and 28 that guarantee non-negative expected payoffs of providers, they might still have negative profit and possibly not able to comply the promise. For that reason, more sensible switching fees should be restricted to non-negative values.

To prove this last proposition, we have to prove that there is no equilibrium where none switches, and there is no equilibrium where everyone switches. The proof is shown in the appendix.

4 Equilibrium analysis: implications

From the symmetric equilibrium conditions given in proposition 3, we can graphically observe the feasible region for switching fees depicted in figure 1. The figures (a) and (b) shows that feasible region for switching fees and price to loyal consumers remain constant across patience level. For first period prices, the upper bound marginally increases with patience level, but the lower bound decreases as the discount factor approaches to one; thus for higher δ, the feasible region of negative prices becomes bigger.



(a) Switching fee s_i (b) Price to loyal consumers T_{ii} (c) First period price T_i

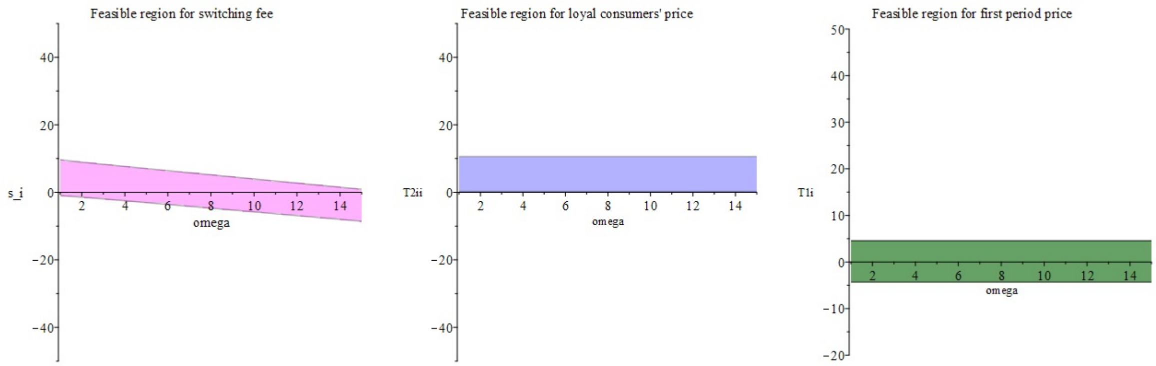
Parameters values: $v = 10$, $\phi = 0.2$ and $\omega = 2$

Figure 1: Feasible regions for switching fees optimal prices as δ changes

In the same fashion, fixing the patience level, we can check that the feasible region for optimal switching fees shifts downwards as the individual's cost of switching effort (exogenous switching cost) parameter increases – both upper and lower bounds decreases in ω–. Meanwhile, the feasible region for prices to loyal consumer remain constant and above zero, and the first period price remains also constant but it allows for negative prices (see figure 2).

Although there are many combinations of prices and switching fees, profits are set in a unique way; providers' profits are increasing in the discount rate and the individual-specific cost of switching effort parameter. Figure 3 shows the feasible regions for first, second and multiperiod profits. Providers may risk and get negative first period profits as discount factor increases. Despite of the multiplicity of equilibrium outcomes for period-profits due to the use of a range of switching fees, the ex-ante multiperiod or lifetime profit is uniquely determined.

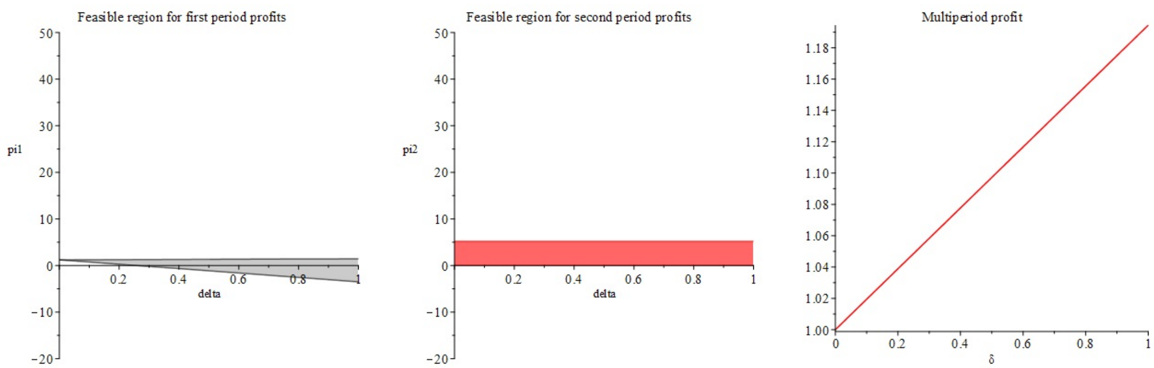
Figures 4 and 5 show the optimal prices and profits as functions of discount factor δ in different scenarios, when providers set minimum and maximum switching fee. Assuming providers always set s^{min}, Figure 4a shows that first-period prices are almost constant and always higher than second-period prices for loyal consumers and switchers; switchers are charged the same regardless of the discount factor.



(a) Switching fee s_i (b) Price to loyal consumers T_{ii} (c) First period price T_i

Parameters values: $v = 10$, $\phi = 0.2$ and $\delta = 0.9$

Figure 2: Feasible regions for switching fees optimal prices as ω changes



(a) First period profits π_{1i} (b) Second period profits π_{2i} (c) Multiperiod profits π_i

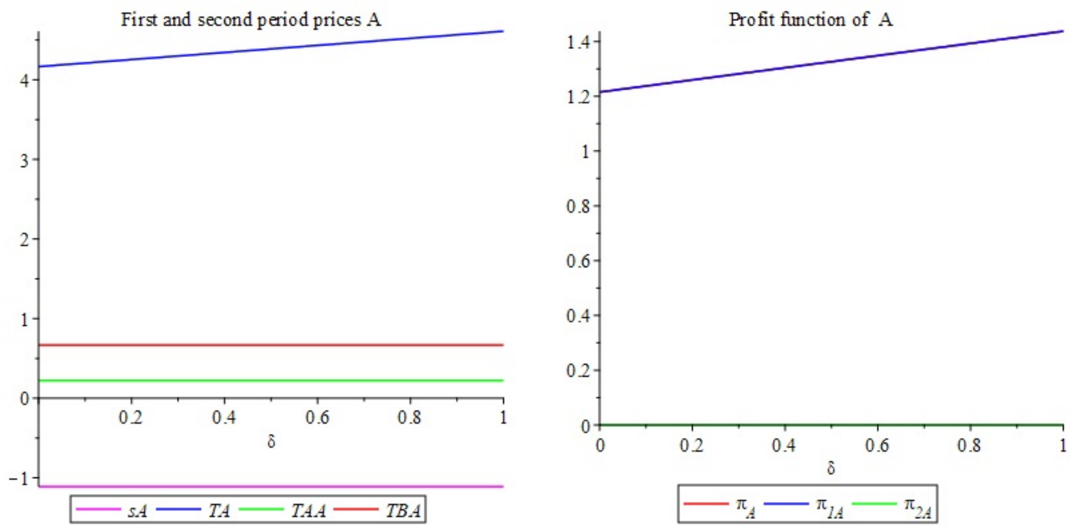
Parameters values: $v = 10$, $\phi = 0.2$ and $\omega = 2$

Figure 3: Feasible regions for optimal first period, second period and multiperiod profits across δ values

Likewise, figure 4b depicts the profit functions: multiperiod profit (red line) is always positive and increasing in δ; first-period profits also are positive but they slightly decrease with patience level. Second-period profits are increasing in δ, but they are negative if $s_i = s^{min}$. This result indicates that the effect of a switching fee is inter-temporally compensated in providers' profits.

When providers set a s^{max}, then second period prices and switching fees are positive, but first period prices quickly becomes negative as discount factor increases. Also first period profit are negative and keep decreasing with patience level, as shown by Figure 5. In this scenario, providers extract the entire consumer surplus in the second period and charge a low (even negative) first-period prices. First period profits also can be negative following the trend of first period prices; despite this, multiperiod profits are kept positive and slightly increasing in δ.

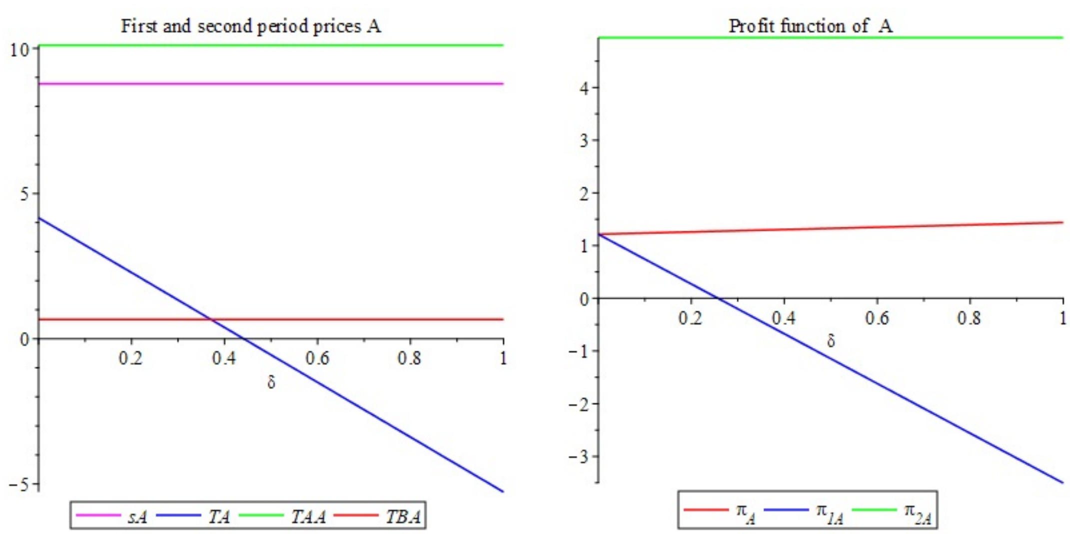
It is important to highlight that the multiperiod profit function in both scenarios is the same, which is explained by the fact that switching fees do not affect multiperiod profits, their effect on period profits are compensated leaving multiperiod profits unaffected.



(a) Optimal per period prices (b) Multiperiod and per period profits

Parameters values: $v = 10$, $\phi = 0.2$ and $\omega = 2$.

Figure 4: Optimal prices and profits when both providers set $s_i = s^{min}$



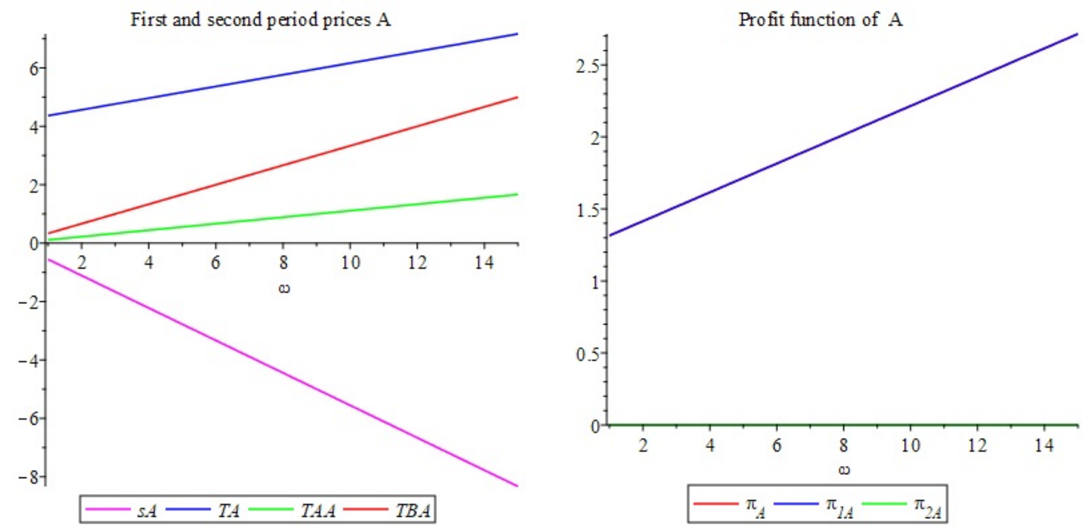
(a) Optimal per period prices (b) Multiperiod and per period profits

Parameters values: $v = 10$, $\phi = 0.2$ and $\omega = 2$.

Figure 5: Optimal prices and profits when both providers set $s_i = s^{max}$

Figures 6 and 7 show the optimal prices and profits as functions of the switching cost parameter ω when providers set minimum and maximum switching fee (a positive amount).[30] It

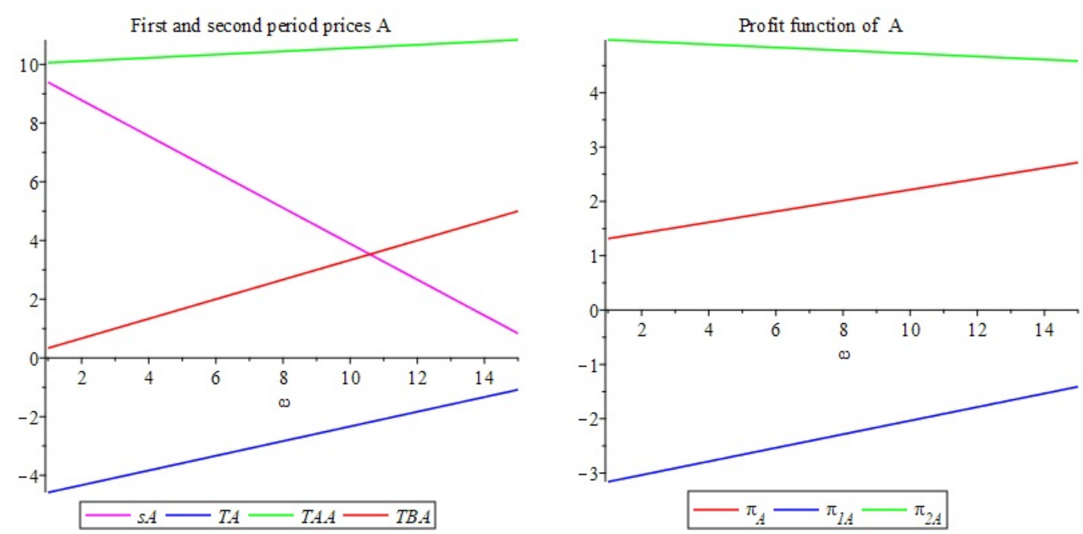
[30]Figures 9 and 10 show the optimal prices and profits as functions of both, δ and ω. In such scenario, first period prices and first period profits are decreasing in patience level and exogenous switching cost parameter ω. Loyals are charged higher than switchers and both prices increase with ω, and multiperiod and second period profits are also increasing in ω, but the latter increases more rapidly.



(a) Optimal per period prices (b) Multiperiod and per period profits

Parameters values: $v = 10$, $\phi = 0.2$ and $\delta = 0.9$.

Figure 6: Optimal prices and profits when both providers set $s_i = s^{min}$



(a) Optimal per period prices (b) Multiperiod and per period profits

Parameters values: $v = 10$, $\phi = 0.2$ and $\delta = 0.9$.

Figure 7: Optimal prices and profits when both providers set $s_i = s^{max}$

is always the case that second period prices increases with ω, while switching fees decreases with ω. When switching fees are set at its minimum (a negative amount), first period prices equals the consumer valuation for the service v, and is independent of exogenous switching costs, when switching fee is set at its maximum, first period prices are negative but increasing in ω.

Ex-ante multiperiod profits are always increasing in exogenous switching costs; when minimum switching fees are set, first period profits reach their maximum, while second period profits are negative but increasing in ω. At maximum switching fees, first period profits are negative (to compensate consumers firms set negative first period prices) but increasing in ω; second period prices are positive but slightly decreasing in ω, this happens due to the effect of lower switching fees collected from more switchers. [31]

Consumer surplus and social welfare

Let's now consider and depict the effect of the equilibrium outcomes, obtained from the extended model, on the consumer surplus and social welfare or total surplus (producer plus consumer surplus). Integrating over consumers, and given $\phi \geq \frac{15}{18v(1+\delta)-5\delta\omega}$, we get the following consumer surplus function:

$$CS = \frac{5\delta(18v - 5\omega)}{108\phi} + \frac{5v}{6\phi} - \frac{25}{48\phi^2} \tag{32}$$

Adding the producer surplus generated by the two providers, and under the allowed range for ϕ, we get the following social welfare function:

$$SW = \frac{5\delta(18v - 5\omega)}{108\phi} + \frac{5v}{6\phi} + \frac{2\delta\omega}{9} + \frac{35}{72\phi} - \frac{25}{48\phi^2} \tag{33}$$

Proposition 5. *If $v \geq \frac{5\omega}{18}\frac{\delta}{1+\delta} + \frac{1}{6\phi(1+\delta)}$, while any switching fees satisfy (27) and (28), in all equilibrium outcomes, the payoffs of consumers and providers remain the same no matter what are the switching fees.*
Additionally, when $\phi \geq \frac{15}{18v(1+\delta)-5\delta\omega}$, consumer surplus always decreases with exogenous switching costs ω; and total surplus (social welfare) decreases with ω only for small marketing cost parameter, $\phi < \frac{25}{24}$.

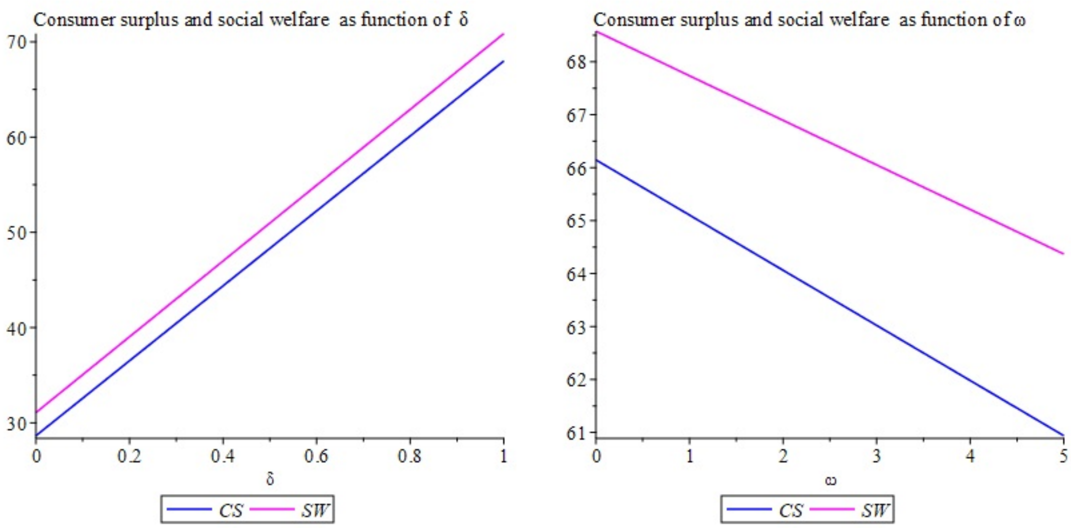
The ability of providers to set switching fees (endogenous switching costs) do not affect the multiperiod payoff of consumers (they affect per period payoff, and these effects that are canceled out in the total discounted multiperiod payoff), therefore multiperiod consumer surplus is also unaffected by the presence of switching fees. However, consumer's multiperiod and per period payoff are affected by exogenous switching costs.

Given that multiperiod profits of providers are also unaffected by the setting of switching fees, social welfare (defined as the summation of consumer surplus and providers' profits) is also unaffected by switching fees (endogenous switching costs). This result may be striking, but it may explain why in some industries such as telecommunications, switching fees like ETF are being dismissed by some companies. It also agrees with the findings of Cullen et al. (2016) where equilibria where providers with and without switching fees may coexist. In the model presented in this paper that may happen because the effect of switching fees are compensated inter-temporally in such a way that they do not affect payoffs of consumers either providers.

Figures 8a and 8b show the consumer surplus (CS) and social welfare (SW) as functions of the discount factor δ, and of the exogenous switching cost parameter ω whenever $\phi \leq 1$. Both functions are clearly increasing in the patience level (δ), driven basically for greater consumer welfare as patience level increases.

On the other hand, consumer surplus decreases more rapidly with the exogenous switching

[31] Recall that the switchers' share rises with ω.



(a) CS and SW as functions of δ (b) CS and SW as functions of ω

Parameters values: $v = 10$, $\phi = 0.2$ and $\omega = 2$.

Figure 8: Consumer surplus and social welfare functions

cost parameter than in the case of social welfare. Thus, less exogenous switching costs would have a greater impact in the short term for consumers.

Table 2: Providers set different switching fee, s^{min}, $s_i = 0$ or s^{max}

	s^{min} vs. s^{max}		A and B set $s_i = 0$	$s_i = 0$ vs. s^{max}		$s_i = 0$ vs. s^{min}	
	A (s^{min})	B (s^{max})	Firm i	A ($s_i = 0$)	B (s^{max})	A ($s_i = 0$)	B (s^{min})
Multiperiod profit π_i	**1.42**	**1.42**	**1.42**	**1.42**	**1.42**	**1.42**	**1.42**
π_{1i}	1.42	-3.03	0.92	0.92	-3.03	0.92	1.42
π_{2i}	0	4.94	0.56	0.56	4.94	0.55	0
T_i	**4.57**	**-4.33**	**3.57**	3.57	**-4.33**	3.57	**4.57**
s_i	-1.11	8.78	0	0	8.78	0	-1.11
T_{ii}	0.22	10.11	**1.33**	1.33	10.11	1.33	0.22
Cost of switching max	1.56	11.44	2.67	2.67	11.44	2.67	1.56
Cost of switching min	-0.44	9.44	0.67	0.67	9.44	0.67	-0.44
R_i	**14.33**	**14.33**	**14.33**	**14.33**	**14.33**	**14.33**	**14.33**
R_{1i}	5.43	**14.33**	**6.43**	6.43	14.33	6.43	5.43
ER_i	**9.89**	**0**	**8.78**	8.78	0	8.78	9.89
$R_{ii} = R_{ij}$	9.78	**-0.11**	8.67	8.67	-0.11	8.67	9.78
CS^i	**32.03**	**32.03**	**32.03**	**32.03**	**32.03**	**32.03**	**32.03**
CS	**64.06**	**64.06**	**64.06**	**64.06**	**64.06**	**64.06**	**64.06**
SW	**66.89**	**66.89**	**66.89**	**66.89**	**66.89**	**66.89**	**66.89**

Parameters values: $\delta = 0.9$, $v = 10$, $\omega = 2$, and $\phi = 0.2$.
Prices to switchers are $T^i_{2j} = 0.67$, and $\theta_1 = \theta_2 = 2.08$ for all the cases.
Cost of switching includes switching fee, maximum (minimum) exogenous switching cost $\omega(0)$ and switcher's price.

By using some numerical exercises; Table 2 presents the different calculated values for ex-ante multiperiod profits (for A and B), ex-ante multiperiod consumer surplus, and ex-ante multiperiod payoff of a typical consumer, as well as per period profits and typical consumer's payoff under different scenarios.

For the same discount factor ($\delta = 0.9$), this table 2 shows that for any combination of maximum or lowest switching fees used by the providers, ex-ante multiperiod payoffs (profits, indirect utilities, and consumer surpluses) are kept unchanged. The observed differences come from the existing trade-off between inter-temporal payoffs when a low or high switching fee is chosen by the providers.

4.1 Discussion and policy implications

The model developed in this paper, show that exogenous switching costs are more relevant than endogenous switching costs in the decision making of consumers. For the providers, switching fees would not affect multi-period profits, but would accentuate a trade off between present and future profits. Providers with high switching fees would compensate consumers with lower first period prices, but would charge higher second period prices to loyal consumers; low switching fees would be associated to high first period prices and lower second period prices to loyals. Thus consumers with lower first period surplus get compensated with higher second period surplus, and *vice versa*.

Second period prices are positively affected by exogenous switching cost parameter ω. Therefore an unanticipated external reduction of exogenous switching costs would reduce second period prices, for both loyal consumers and switchers; however, if the change is anticipated for the providers, this reduction also would increase first period prices and possibly leads to higher switching fees.

On the other hand, since profits are increasing in exogenous switching costs ω, providers will have incentives to keep a high ω (opposing to regulatory changes such number portability or standardization or even by increasing searching costs). Also, given that profits are increasing in relative taste parameters, providers have greater incentives to invest in advertising to influence consumer preferences, when they do, in a symmetric case, firms invest until they both get same relative taste level.

According to the model, firms charge higher to loyal consumers than to newcomers in the second period when a maximum switching fee is charged, and otherwise if the minimum switching fee is applied. Furthermore, when s^{max} is used by both providers, then these charge higher to loyal consumers in the second period respect to first period prices.

Once again, switching fees do not play any role in total discounted payoffs (profits and consumer surplus). The negative effect of switching fees on first period profits cancels out with the positive effect it has in the second period profits. This would explain why we observe some competing providers that dismiss switching fees from their pricing strategy, while others maintain it. This result is also consistent with the findings of Cullen et al. (2016).

Hence, policies that target exogenous switching costs reduction may have higher impact on social welfare than those that ban any existence of switching fees (endogenous SC); external reduction of exogenous switching costs increases social welfare, by increasing consumer surplus.

The model suggests that regulatory policies that reduce exogenous switching costs such as number portability (in telecommunication industries, or banking industries), compatibility, standardization, or reduction of administrative barriers, would be more effective in increasing social welfare than policies that reduce endogenous switching costs such as switching fees (ETF in telecommunication industry), because the expected outcomes (payoffs) remain the same despite variations of switching fees.

5 Conclusions

The model developed in this paper shows that exogenous switching costs (individual-specific costs of switching efforts) are more relevant than endogenous switching costs (switching fees) in the decision making of consumers. For the providers, switching fees would not affect ex-ante multiperiod profits but would accentuate a trade-off between present and future profits. Providers with high switching fees would compensate consumers with lower first period prices, but would charge higher second-period prices to loyal consumers; low switching fees would be associated with high first-period prices and lower second-period prices to loyals. Thus consumers with lower first-period surplus get compensated with a higher second-period surplus and vice versa.

Second-period prices are positively affected by individuals' costs of switching effort parameter ω. Therefore an unanticipated external reduction of exogenous switching costs would reduce second-period prices, for both loyal consumers and switchers; however, if the providers anticipate the change, this reduction also would increase first-period prices and the possibility of higher switching fees. However, since the adverse effect of switching fees on first-period profits cancels out with their positive effect on the second-period profits, then regulatory policies should focus more on policy measures that reduce individuals' costs of switching effort by promoting standardization, compatibility, number portability, or speeding and easing the switching process (red-tape reduction).

On the other hand, since ex-ante multiperiod profits are increasing in the individual' cost of switching efforts (exogenous switching costs) ω, therefore providers will have incentives to keep a high ω (opposing to regulatory changes such number portability or standardization or even by increasing searching costs). However, high individual switching costs induce firms to price very low or even negative in the first period to attract consumers, despite of charging a maximum switching fee; first period profits are decreasing in exogenous switching costs.

According to the model, providers charge higher to loyal consumers than to newcomers in the second period when patience level is high. When both providers charge a maximum switching fee, then they charge higher to loyal consumers in the second period respect to first-period prices.

The effect of switching fees in ex-ante multiperiod payoffs is null, in other words, switching fees are neutral. Hence policies that target exogenous switching costs reduction may have a higher impact on social welfare than those that ban any existence of switching fees (endogenous SC); external reduction of individual switching costs increases social welfare, by increasing consumer surplus.

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Appendix

Expected second period consumer surplus

In the first period, consumers make a choice between providers, therefore, the payoff of a consumer k will be given by :

$$R_{1k}^A = v + \sigma_k - T_A$$
$$R_{1k}^B = v + \sigma_k - T_B$$

where σ_k is the relative preference for firm A respect to firm B , and is uniformly distributed on the interval $[-\theta_1, \theta_2]$.

However in the first period, consumers do not take decisions only based on their current period payoffs, but based on their multiperiod payoffs. Thus, each consumer compare R^A vs. R^B

$$R^A = R_1^A + \beta E_x[R_{2A}]$$
$$R^B = R_1^B + \beta E_x[R_{2A}]$$

where

$$E[R_{2i}] = P_{ii}R_{ii} + P_{ij}R_{ij} \qquad \forall i, j \in \{A, B\}$$

Therefore, we get $E_x[R_{2A}]$ and $E_x[R_{2B}]$ using the distribution of exogenous switching costs x_k

$$E_x[R_{2A}] = v - \left(\int_{x_A}^{\omega} T_{AA}^* \frac{1}{\omega} dx + \int_0^{x_A} (T_{AB}^* + s_A + x) \frac{1}{\omega} dx \right) = v - \tfrac{11}{18}\omega - s_A$$

$$E[R_{2B}] = v - \left(\int_{x_B}^{\omega} T_{BB}^* \frac{1}{\omega} dx + \int_0^{x_B} (T_{BA}^* + s_B + x) \frac{1}{\omega} dx \right) = v - \tfrac{11}{18}\omega - s_B$$

Therefore,

$$R_A = v + \sigma - T_A + \beta(v - \tfrac{11}{18}\omega - s_A)$$
$$R_B = v - \sigma - T_B + \beta(v - \tfrac{11}{18}\omega - s_B)$$

Proofs

Proof Proposition 1:
No firm will profitable deviate from the equilibrium prices.

Proof. Suppose provider A deviates and use prices $\widehat{T_{AA}}$ and T_{BA}^* , where $\widehat{T_{AA}} = T_{AA}^* + \Delta$, while provider B keep using equilibrium prices T_{BB}^* and T_{AB}^*. We can check, using (8) that new profits of provider A after deviation are

$$\begin{aligned}
\widehat{\pi_{2A}} &= \alpha \widehat{T_{AA}} - \frac{\alpha}{\omega}(\widehat{T_{AA}} - T_{AB}^* - s_A^*)(\widehat{T_{AA}} - s_A^*) + \frac{(1-\alpha)}{\omega} T_{BA}^*(T_{BB}^* - T_{BA}^* - s_B^*) \\
&= \pi_{2A}^* + \frac{\alpha}{\omega}[\Delta\omega - \Delta(T_{AA}^* - T_{AB}^* - s_A^*) + \Delta(T_{AA}^* - s_A^*) - \Delta^2] \\
&= \pi_{2A}^* - \frac{\alpha\Delta^2}{\omega}
\end{aligned}$$

Then, $\widehat{\pi_{2A}} < \pi_{2A}^*$.

Now, *ceteris paribus*, suppose A deviates to $\widehat{T_{BA}} = T^*_{BA} + \Delta$; then in similar fashion and using (8) that new profits are

$$\begin{aligned}
\widehat{\pi_{2A}} &= \alpha T^*_{AA} - \frac{\alpha}{\omega}(T^*_{AA} - T^*_{AB} - s^*_A)(T^*_{AA} - s^*_A) + \frac{(1-\alpha)}{\omega}\widehat{T_{BA}}(T^*_{BB} - \widehat{T_{BA}} - s^*_B) \\
&= \pi^*_{2A} + \frac{1-\alpha}{\omega}[-\Delta T^*_{BA} + \Delta(T^*_{BB} - T^*_{BA} - s^*_B) - \Delta^2] \\
&= \pi^*_{2A} - \frac{\alpha\Delta^2}{\omega}
\end{aligned}$$

Once again, $\widehat{\pi_{2A}} < \pi^*_{2A}$.

Therefore, regardless of the deviation ($\Delta > 0$ or $\Delta < 0$), profits are always lower than the profit achieved with equilibrium prices, and providers do not have any profitable deviation. □

Proof Proposition 4

Claim 1: There is no equilibrium where nobody switches.

Proof. Let's suppose $x_A > \omega$ & $x_B > \omega$ and analyze the game in the second period. In this case, consumers prefer to stay with their provider, which means that the payoffs of a consumer that chose A in the first period are as follows:

$$\begin{aligned}
R_{AA} \geq 0 \quad &\Rightarrow \quad v \geq T_{AA} \\
R_{AB} \leq 0 \quad &\Rightarrow \quad v - s_A - x \leq T_{AB}
\end{aligned}$$

Likewise, the payoff of a consumer that chose B in the first period are

$$\begin{aligned}
R_{BB} \geq 0 \quad &\Rightarrow \quad v \geq T_{BB} \\
R_{BA} \leq 0 \quad &\Rightarrow \quad v - s_B - x \leq T_{BA}
\end{aligned}$$

Since consumers are better off staying than switching, then $R_{AA} \geq R_{AB}$ and $R_{BB} \geq R_{BA}$. Therefore the following must hold:

$$\begin{aligned}
T_{AB} + s_A + x \geq T_{AA} \\
T_{BA} + s_B + x \geq T_{BB}
\end{aligned}$$

Given consumers preferences, providers set their second period prices that maximize their profits assuming the rival provider charges zero to newcomers; thus $T_{ii} > 0$ to loyal consumers and $T_{ji} = 0$ for $i \neq j \;\; i, j \in \{A, B\}$ to rival's consumers. Therefore, firm A solves the following problem:

$$\max_{T_{AA}} \pi_{2A} = \alpha T_{AA}$$
$$s.t. \quad \substack{R_{AA} \geq 0 \\ R_{AA} \geq R_{AB} \\ T_{BA} = 0 \\ x \sim U[0,\omega]}$$

which is reduced to the following:

$$\max_{T_{AA}} \pi_{2A} = \alpha T_{AA}$$
$$s.t. \quad T_{AA} \leq min\{v, s_A + x^{min}\}$$

Given the distribution of x, then $x^{min} = 0$. Also, v is the reservation value of any consumer. By construction, $v \leq s_A + x$, therefore, s_A cannot be lower than v. Thus, since profits are increasing in T_{AA}, providers will price as high as possible, which means the maximizing price T_{AA} for firm A is v.

$$T^*_{AA} = v$$

Similarly for firm B, then

$$T^*_{BB} = v$$

Therefore, providers' profits in the second period are given by:

$$\begin{aligned} \pi^*_{2A} =& \alpha v \\ \pi^*_{2B} =& (1-\alpha)v \end{aligned}$$

Now, suppose firm A, *ceteris paribus*, increase its price T^*_{AA} to $\widehat{T_{AA}} = v + \epsilon \quad \forall \epsilon \in (0, \frac{\omega}{2})$. Since it is increasing its price a bit (by ϵ), there will be some consumers that switch. We can check that by looking at the preferences and payoffs of consumers. At the new price $\widehat{T_{AA}}$, consumers will stay when $R_{AA} \geq R_{AB}$, i.e.

$$\begin{aligned} v - (v + \epsilon) \geq v - 0 - v - x \\ x \geq \epsilon \end{aligned}$$

thus, provided that $x \in [0, \omega]$, the new choice probabilities are:

$$\begin{aligned} \widehat{P_{AA}} &= \int_{\epsilon}^{\omega} \frac{1}{\omega} dx = \frac{\omega - \epsilon}{\omega} \\ \widehat{P_{AB}} &= \int_{0}^{\epsilon} \frac{1}{\omega} dx = \frac{\epsilon}{\omega} \end{aligned}$$

And the shares of loyal consumers to A and switchers from A to B are $n_{AA} = \alpha\widehat{P_{AA}}$ and $n_{AB} = \alpha\widehat{P_{AB}}$, respectively. Then, new profits become:

$$\begin{aligned} \widehat{\pi_2^A} &= \alpha(1 - \frac{\epsilon}{\omega})(v + \epsilon) + \alpha\frac{\epsilon}{\omega}v \\ &= \alpha v + \frac{\alpha\epsilon}{\omega}(\omega - \epsilon) \\ &= \pi_2^{A*} + \frac{\alpha\epsilon}{\omega}(\omega - \epsilon) \end{aligned}$$

Thus, since $\epsilon < \omega$ by construction, firm A would deviate to $\widehat{T_{AA}}$, increasing its price and getting higher profits ($\widehat{\pi_2^A} > \pi_2^{A*}$). Therefore, there is no an equilibrium where nobody switches. $\square$

<u>*Claim 2*</u>: There is no equilibrium where everyone switches.

Proof. Let's suppose $x_A < 0 \;\; \& \;\; x_B < 0$ and as before, I analyze the game in the second period. In this case, consumers prefer to switch rather than stay with their provider, which means that the payoff of a consumer that chose A in the first period are as follows:

$$\begin{aligned} R_{AA} \leq 0 \;\; &\Rightarrow \;\; v \leq T_{AA} \\ R_{AB} \geq 0 \;\; &\Rightarrow \;\; v - s_A - x \geq T_{AB} \end{aligned}$$

Likewise, the payoff of a consumer that chose B in the first period are

$$\begin{aligned} R_{BB} \leq 0 \quad &\Rightarrow \quad v \leq T_{BB} \\ R_{BA} \geq 0 \quad &\Rightarrow \quad v - s_B - x \geq T_{BA} \end{aligned}$$

Since consumers are better off switching than staying, then $R_{AA} \leq R_{AB}$ and $R_{BB} \leq R_{BA}$. Therefore the following must hold:

$$\begin{aligned} T_{AA} - s_A - x \geq T_{AB} \\ T_{BB} - s_B - x \geq T_{BA} \end{aligned}$$

Given the preferences of consumers, providers set their second period prices that maximize their profits assuming the rival provider charges zero to their consumers in order to retain them; thus $T_{ii} = 0 \ \forall i \in \{A, B\}$.

Therefore, firm A solves the following problem:

$$\max_{T_{BA}} \pi_{2A} = (1 - \alpha)T_{BA} + \alpha s_A$$
$$s.t. \quad {\scriptstyle \substack{R_{BA} \geq 0 \\ R_{BB} \leq R_{BA} \\ T_{BB} = 0 \\ x \sim U[0, \omega]}}$$

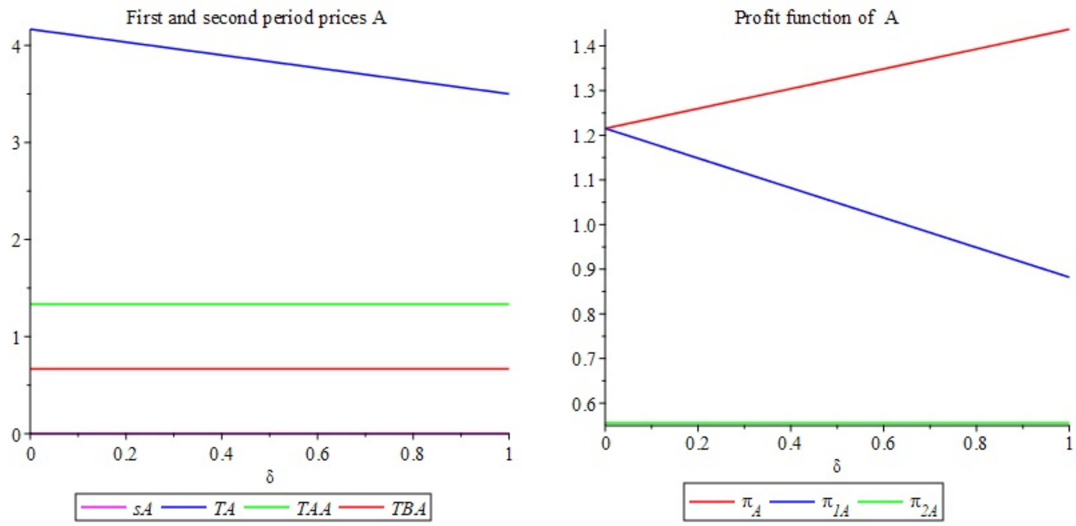
which is reduced to the following:

$$\max_{T_{BA}} \pi_{2A} = (1 - \alpha)T_{BA} + \alpha s_A$$
$$s.t. \quad {\scriptstyle T_{BA} \leq min\{v - s_B - x^{max}, -s_B - x^{max}\}}$$

Given the distribution of x, then $x^{max} = \omega$. Recall that provider A charges $T_{AA} = 0$, which imply zero reservation value of consumers for the service, $v = 0$ because this value cannot be negative. Therefore, $T_{BA} = -s_B - \omega$, providers would make losses in the second period. Also, since reservation value of consumers does not change between periods, consumers would not be interested in buying the service if the first period prices are positive, recall that $v = 0$. Thus, providers would need to price zero in both periods, and finally they would just make losses by operating under this case, therefore providers would be better off by not operating. Hence, there is not an equilibrium where everyone switches. $\square$

Given that we claim 1 and 2 are true, we proved proposition 4.

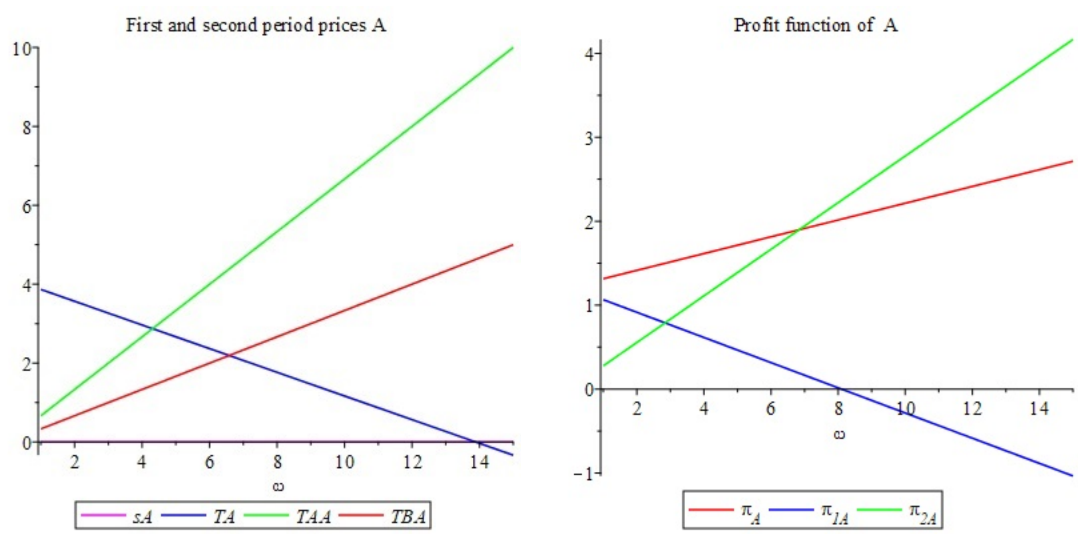
5.1 *Optimal prices and profit functions when providers set $s_i = 0$



(a) Optimal per period prices (b) Multiperiod and per period profits

Parameters values: $v = 10$, $\omega = 2$, and $\phi = 0.2$.

Figure 9: Optimal prices and profits as functions of δ, when both providers set $s_i = 0$



(a) Optimal per period prices (b) Multiperiod and per period profits

Parameters values: $v = 10$, $\omega = 2$, and $\phi = 0.2$.

Figure 10: Optimal prices and profits as functions of ω, when both providers set $s_i = 0$